G-10/2051 5821/MJ

Mathematical Methods-403

Sem-IV

Syll-May-2018

Time - 3hrs M.M.-75

Note: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

SECTION A

1. State and prove Rodrigue's formula.
2. Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x^2}(3-x^2)\sin x - \frac{3}{x}\cos x\right]$.
3. Find the Eigen functions of the following Sturm-Liouville problems and verify their orthogonality $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = y(2\pi), y'(0) = y'(2\pi)$
4. Expand $f(x) = x^3 + x, -1 \le x \le 1$ in terms of Chebyshev polynomials of second kind $U_n(x)$.

$2 \times 11 = 22\frac{1}{2}$

SECTION B

5. Apply the Convolution theorem to prove that $\int_0^t \sin u \cos(t-u)\, du = \frac{t}{2}\sin t$.
6. Find the Fourier series expansion of the following period function of period 2π

$$f(x) = \begin{cases} 0 & ; -\pi < x < 0 \\ x^2 & ; 0 \le x < \pi \end{cases}.$$

Hence, show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$$

7. Find the inverse Laplace transform of the function $\frac{s}{(s^2+a^2)^3}$.
8. Show that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt = \log\frac{2}{3}$ by using the Laplace transformation.

$2\frac{1}{4} \times 11 = 22\frac{1}{2}$

SECTION C

9. Write briefly
   - a) State the necessary and sufficient conditions of Laplace Transform.
   - b) State the orthogonal property of Bessel function and Chebyshev polynomial of first kind.
   - c) Find the Laplace transform of t^2e^t.
   - d) State Convolution theorem.
   - e) State Sturm-Liouville problem.
   - f) Write Euler's formula for Fourier series.
   - g) Show that $P_n'(1) = n(n+1)/2$.
   - h) Define Generating function of the Legendre polynomial.
   - i) Express the function $f(t) = \begin{cases} 0 & ; 0 \le t < 3 \\ (t-3)^2 & ; t \ge 3 \end{cases}$ in terms of unit step functions and hence evaluate its Laplace transform.
   - j) Show that $P_n(-x) = (-1)^n P_n(x)$.

$10 \times 3 = 30$