PC-729/MH

BS/2051
DYNAMICS–VI
(Semester–IV)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B Section C will be compulsory.

SECTION–A

I. A particle projected vertically upwards takes time t_1 to reach a height h. If t_2 is the time to reach the ground again, prove that $h = \frac{1}{2} g\, t_1 t_2$ and the maximum height attained is $\frac{1}{8} g\, (t_1 + t_2)^2$. Also, find the velocity of the particle at a height $\frac{h}{2}$.

II. At the end of three successive seconds, the distance of a point moving with simple harmonic motion from its mean position measured in the same direction are 1, 5, 5. Show that the period of complete oscillation is $\frac{2\pi}{\theta}$ seconds where $\cos\, \theta = \frac{3}{5}$.

III. A bullet travelling horizontally pierces successively three screens places at equal distance b apart. If the time from the first screen to the second is t_1 and from the second to third is t_2, prove that the retardation, assumed to be constant, is $\frac{2b(t_1 - t_2)}{t_1\, t_2(t_1 + t_2)}$.

IV. A particle is projected upwards with a velocity of u m/sec and after t seconds another particle is projected upwards from the same point and with the same velocity. Prove that the particles meet at height $\frac{4u^2 - g^2t^2}{8g}$ meters after a time $\left(\frac{t}{2} + \frac{u}{g}\right)$ seconds. (2×6=12)

SECTION–B

V. A train of mass M kg is ascending an incline of 1 in n, the resistance of motion being m kg weight per kg. of weight of train. The speed of the train is v metres/sec, when the power developed in the engine is H watts. Show that the acceleration is given by $\frac{n\,\mathrm{H} - v\,\mathrm{M}\,g(1 + mn)}{\mathrm{M}\,nv}$ m/sec^2.

VI. A uniform elastic string has length a_1 when the tension is T_1 and length a_2 when the tension is T_2. Show that its natural length is $\frac{a_2T_1 - a_1T_2}{T_1 - T_2}$ and the amount of work done

in stretching it from its natural length to a length $a_1 + a_2$ is

$$\frac{(a_1T_1 - a_2T_2)^2}{2(T_1 - T_2)(a_1 - a_2)}.$$

VII. A 250 kg gun fires a shot weighing 2 kg. if the recoil apparatus exerts a constant force of 100 newtons and the gun moves back 0.8 meters. Calculate the muzzle speed of the shot.

VIII. A shell of mass *m* is projected from a gun of mass M by an explosion which generates kinetic energy E. Prove that the initial velocity of the shell is $\sqrt{\frac{2EM}{m(M+m)}}$, it is assumed that gun is free to recoil. (2×6=12)

SECTION–C

IX. Attempt all the questions :

(a) Define Work, Power and Energy.

(b) Find how many seconds a clock would lose per day if the length of its pendulum is increased in the ration of 900 : 901.

(c) A particle is performing S.H.M. between two points A and B. If the period of oscillation is 2π, show that the velocity at any point P is mean proportional between AP and BP.

(d) Define simple harmonic motion and find its amplitude.

(e) Show that angular momentum of a satellite of mass m which over round the earth is $m\sqrt{GMr}$ where r is the radius of the circular orbit is, M is the mass of the earth and G is the universal constant.

(f) Prove that tension in the string is the harmonic mean between the weights of the two bodies.

(g) Compute the angular momentum of Neptune about the sun given that it moves in a circular orbit of radius 5×10^{12} meter. Mass of Neptune is 10^{26} kg and its completes one revolution in 165 years.

(h) If the time of one complete oscillation of a single pendulum is 20 seconds, find the length of the pendulum. (8×2=16)