PC-747/MH

CS/2051

DISCRETE MATHEMATICS–II

Paper–IV

(Semester–VI)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION–A

I. (a) Determine the generating function of the numeric function a_r, where

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases} \quad (3)$$

(b) Determine the discrete numeric function corresponding to the following generating function is

$$A(z) = \frac{z^5}{5 - 6z - z^2}. \quad (3)$$

II. (a) Solve the recurrence relation $a_r = 2a_{r-1} + 3$, $r \geq 1$. With $a_0 = 1$, by the method of generating function. (3)

(b) Solve the following recurrence relation by Characteristic Roots Method

$a_r = a_{r-1} + a_{r-2}$, $r \geq 2$. with $a_0 = 1$, $a_1 = 1$. (3)

III. Solve the following equation for homogeneous and particular solution

$$a_r - 4a_{r-1} + 4a_{r-2} = f(r),$$

where $f(r) = 3r + 2^r$ with $a_0 = a_1 = 1$. (6)

IV. (a) What do you mean by Complexity of Problems. (3)

(b) Design an algorithm to select the largest and second largest of n numbers. The basic operation is to compare two numbers and determine the larger and smaller of the two. (3)

SECTION–B

V. (a) Simplify the Boolean expression

$f(x, y, z) = (\overline{x} \wedge z) \vee (y \wedge z) \vee (y \wedge \overline{z})$ and write in min term normal form. (3)

(b) Let (L, $\leq$) be a lattice. For any elements $a, b \in$ L, prove that $a \wedge b = a \Leftrightarrow a \vee b = b$. (3)

VI. (a) Prove that the following are equivalent in a Boolean Algebra :

(i) $a + b = b$.

(ii) $a.b = a$.

(iii) $a' + b = 1$.

(iv) $a.b' = 0$. (3)

(b) Show that the Boolean functions $f_1 = (x_1 \vee x_2) \vee x_3$ and $f_2 = x_1 \vee (x_2 \vee x_3)$ are equivalent. (3)

VII. (a) Prove that the set D_n of all positive divisors of n is a bounded distributive lattice. (3)

(b) Prove the validity of following arguments :

If man is a bachelor, he is unhappy.

If a man is unhappy, he dies young.

Therefore, bachelor die young.

VIII. (a) Prove that a group homomorphism $f : G \to G'$ is one-one if and only if Ker $f = \{e\}$. (3)

(b) Prove that $p \wedge q = q \wedge p$. (3)

SECTION–C

IX. (a) Define generating function of a sequence.

(b) What do you mean by Fibonacci Numbers?

(c) Find the general solution of recurrence relation.

$$S_n - 3S_{n-1} + 2S_{n-2} = 0.$$

(d) Write a short note on Recurrence relations.

(e) Define Lattice.

(f) Express the output Y as a Boolean expression in the inputs A, B, C for the logic circuit

$$Y = AB' + B'C.$$

(g) Prove that the following are equivalent :

$$p \rightarrow q \Leftrightarrow (\sim q \rightarrow \sim p).$$

(h) Define Boolean Function. (2×8=16)