PC-748/MH

CS/2051

MATHEMATICAL METHODS–II (Opt. iii)

(Semester–VI)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B. Section C will be compulsory.

SECTION–A

I. Find Fourier Transform of $f(t) = \begin{cases} 1, & |t| < \alpha \\ 0, & |t| > \alpha \end{cases}$ and

Hence find $\int_0^\infty \frac{\sin \alpha s \cos st}{s} ds$ and $\int_0^\infty \frac{\sin s}{s} ds.$ (6)

II. (a) Find Fourier sine and cosine transform of

$$f(x) = \begin{cases} 2t, & 0 \le t \le a \\ 0, & t > a \end{cases}$$ (3)

(b) Using Modulation theorem find Fourier cosine transform of

$$f(t) = \begin{cases} 2\cos 3t, & 0 \le t \le a \\ 0, & t > a \end{cases}$$ (3)

III. (a) Use Parseval's identity for Fourier Cosine Transform

show that $\int_0^\infty \frac{dt}{(t^2+1)^2} = \frac{\pi}{4}$. (3)

(b) Find a function $f(t)$ whose fourier cosine transform is

$$\frac{\sin \lambda s}{s}.$$ (3)

IV. (a) Find finite Fourier sine transform of $f(t) = 2, 0 < t < \pi$. Apply inversion formula to find Fourier sine series for $f(t)$ and then evaluate $1 - \frac{1}{3} + \frac{1}{5} - \ldots$ (3)

(b) Find finite Fourier sine transform of $f(t) = t^2$ where $0 < t < \pi$. (3)

SECTION–B

V. (a) Solve the following initial value problem

$X'' + x' = t, \quad x'(0) = 1, \quad x(1) = 0.$ (3)

(b) Solve $tY'' + (1 - 2t)Y' - 2Y = 0,$

$Y(0) = 1, \; Y'(0) = -2.$ (3)

VI. Solve $x' + 5x - 2y = t, \; y' + 2x + y = 0, \; x(0) = 0, \; y(0) = 0.$ (6)

VII. Use complex form of Fourier series to solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, \text{t} > 0 \text{ where } u = f(x) \text{ when } t = 0.$$

(6)

VIII. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{1}{3}\frac{\partial u}{\partial t}$ where $u\left(\frac{\pi}{2}, t\right) = 0$, $u_x(0, t) = 0$, $u(x, 0) = 60 \cos 5x$. (6)

SECTION–C

IX. (a) Define Dirichlet's conditions.

(b) State Fourier Integral formula.

(c) State Parseval's identity for Fourier sine transform.

(d) Find Fourier cosine transform of e^{-t^2}.

(e) Solve :

$(D^4 - a^4)\, y = 0,\ y(0) = 1,\ y'(0) = y''(0) = y'''(0) = 0.$

(f) Solve the heat conduction problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ where

(i) $u(0, t) = 0.$

(ii) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x, t)$ is bounded.

(g) Solve $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}$, $0 < x < 1$, $t > 0$, given that $y(x, 0) = x$.

(h) Solve the integral equation

$$\int_0^\infty f(t) \cos st \, dt = \begin{cases} 1 - s, & 0 \le s \le 1 \\ 0, & s > 1 \end{cases}$$ (8×2=16)
