

Roll No. ....

Total Pages : 5

**1004/MJ**

**F-7/2051**

**PROBABILITY THEORY-II**

Paper-124

Semester-II

Time allowed : 3 Hours] [Maximum Marks : 30

**Note:** The candidates are required to attempt two questions each from section A and section B carrying 4 marks each and the entire section C consisting of 7 questions carrying 2 marks each.

**SECTION-A**

1. A box contains 100 cellphones, 20 of which are defective. 10 cellphones are selected for inspection. Find the probability that :
  - (i) at least one is defective
  - (ii) at most three are defective. 4

2. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.5. Calculate the proportion of days on which :
  - (i) neither car is used
  - (ii) some demand is refused. 4
3. A fast food chain finds that the average time customers have to wait for service is 45 seconds. If the waiting time can be treated as an exponential random variable, what is the probability that a customer will have to wait more than 5 minutes given that already he waited for 2 minutes? 4
4. The weekly wages of 1000 workers are normally distributed around a mean of \$500 with a standard deviation of \$50. Estimate the number of workers, whose weekly wages will be :
  - (i) between \$400 and \$600

(ii) less than \$400

Given that  $P(0 < Z < 2) = 0.4772$ ;  $P(Z < 0.4) = 0.6554$ ;  
 $P(Z < -0.6) = 0.2743$ . 4

### SECTION-B

5. The joint density function of X and Y is given by :

$$p(x, y) = k(2x + y); x = 0, 1, 2; y = 0, 1, 2$$

Find :

(i) value of k

(ii) Marginal distribution of X and Y

(iii) Are X & Y independent variables? 4

6. A random variable X has a mean 8 and variance 9 and an unknown probability distribution. Find  $P(-4 < X < 20)$  and  $P(|X - 8| < 6)$ . 4

7. Examine if WLLN holds for the sequence  $\{X_n\}$  of i.i.d.r.v's with : 4

$$P[X_i = (-1)^{k-1} k] = \frac{6}{2k^2}; k = 1, 2, 3, \dots; i = 1, 2, 3$$

8. Let  $\{X_i\}$  be independent and identically distributed random variables with mean 3 and variance  $\frac{1}{2}$ . Use Central limit theorem to Estimate  $P(340 < S_n < 370)$ , where :

$$S_n = X_1 + X_2 + \dots + X_n \text{ and } n = 120.$$

Given that  $P(Z < -2.58) = 0.0049$ ;  $P(Z < 1.29) = 0.9015$ . 4

### SECTION-C

9. Write in brief:  $2 \times 7 = 14$

(i) Does there exist a variate X for which :

$$P(-2 < X < 2) = 0.6?$$

(ii) State Central Limit theorem.

(iii) Suppose that the probability for an applicant for a driver's licence to pass the road test on any given attempt is  $\frac{2}{3}$ . What is the probability that the applicant will pass the road test on the third attempt?

- (iv) State Geometric distribution.
- (v) Write the p.d.f. of chi-square distribution.
- (vi) State the weak law of large numbers.
- (vii) If the probability of having a male child is 0.5, find the probability that in a family, the eighth child is the third boy.