PC-1031/MJ

F-10/2051

CALCULUS-II-201

(Semester–II)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *two* questions each from Section A and B. Section C will be compulsory.

SECTION–A

I. (a) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + 2y^2}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0) \end{cases}$$

is not continuous at (0, 0).

(b) Find T, N, B, k and τ for the space curve $(e^t \cos t)i + (e^t \sin t)j + 2k$.

II. (a) If $z = xf(x + y) + yg(x + y)$, show that

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(b) If $x = e^{r\cos\theta} \cos (r \sin \theta)$ and $y = e^{r\cos\theta} \sin \theta\ (r \sin \theta)$

prove that $\dfrac{\partial x}{\partial r} = \dfrac{1}{r}\dfrac{\partial y}{\partial \theta}, \ \dfrac{\partial y}{\partial r} = \dfrac{-1}{r} \cdot \dfrac{\partial x}{\partial \theta}$.

III. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $2\hat{i} + 3\hat{j} + 6\hat{k}$.

(b) Find the directional derivative of $\phi = e^{2x} \cos yz$ at the origin in the direction of the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$.

IV. Using Lagrange's multiplier method find the point on the surface of $z = x^2 + y^2 + 10$ nearest to the plane $x + 2y - z = 0$. (2×10=20)

SECTION–B

V. (a) Show that $\int_{(1,2)}^{(3,4)} (xy^2 + y^3)dx + (x^2 y + 3xy^2)dy$ is independent of the path joining the points (1, 2) and (3, 4). Hence evaluate the integral.

(b) Evaluate

$$\iint_S \vec{A} \cdot \hat{n}\, ds, \text{ where } \vec{A} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$$

and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

VI. State and prove Divergence theorem of Gauss.

VII. (a) Evaluate $\iint r \sin\theta \, dr d\theta$ over the area of the cardioid $r = a(1 + \cos\theta)$ above the initial line.

(b) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz dx dy$.

VIII. (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(b) Find the centre of gravity of the area between $y = 6x - x^2$ and $y = x$.

SECTION–C

IX. (a) Find the moment of inertia about x-axis of the arc of the parabola $y = \sqrt{x}$ lying betwee (0, 0) and (4, 2) when $\rho = \dfrac{3M}{16}$ and M is the mass of the arc OP.

(b) Evaluate $\iint xy(x + y) dxdy$ over the area between $y = x^2$ and $y = x$.

(c) Evaluate $\int_0^1 \int_{4y}^4 e^{x^2} \, dxdy$ by changing the order of integration.

(d) Find the normal vector and equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at the point (3, 4, 5).

(e) Find the equation of a plane through $P_0(0, 2, -1)$ normal to $n = 3\hat{i} - 2\hat{j} - k$.

(f) Identify the surface $x = y^2 - z^2$ and draw its rough sketch.

(g) If $r(t) = (3t+1)\hat{i} + \sqrt{3}t\hat{j} + t^2\hat{k}$ is the position of a particle in space at time t. Find the angle between the velocity and acceleration vectors at time $t = 0$.

(h) Find a spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$.

(i) Find the point on the curve

$$r(t) = (12 \sin t)\hat{i} - 12(\cos t)\hat{j} = 5t\,\hat{k}$$

at a distance 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.

(j) Find the local extreme values of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6.$$ (10×3=30)