

A/2051

## Paper-IV: CALCULUS-II

4301/MH

**Note:** Attempt five questions in all selecting two questions from each of the sections A and B and compulsory question of section C.

Max.Marks:40

## Section -A

Q.1. Change into polar co-ordinates and evaluate

$$\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dy dx$$

06

Q.2. Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $y + z = 4$  and  $z = 0$ . 06

Q.3. Find the moment of Inertia of a quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  of mass M about the z-axis, if the density at a point is proportional to  $xy$ . 06

Q.4. Evaluate  $\iiint \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ , the integral being extended to the positive octant of the sphere  $x^2 + y^2 + z^2 = 1$ . 06

## Section-B

Q.5. Verify divergence theorem for  $\vec{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ , over the cylindrical region bounded by  $x^2 + y^2 = a^2$ ,  $z = 0$  and  $z = h$ . 06

Q.6. Evaluate

$$\iint_S \vec{A} \cdot \hat{n} dS$$

06

Where  $\vec{A} = 12x^2y\hat{i} - 3yz\hat{j} + 2z\hat{k}$  where S is the portion of the plane  $x + y + z = 1$  included in the first octant.

Q.7. Show that the integral given below is independent of the path joining the points (1,2) and (3,4). Hence evaluate the integral 06

$$\int_{(1,2)}^{(3,4)} (xy^2 + y^3) dx + (x^2y + 3xy^2) dy$$

Q.8. If  $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ . Evaluate  $\iiint \nabla \times \vec{F} dV$ , where V is the region closed bounded by the planes  $x = 0, y = 0, z = 0, 2x + 2y + z = 4$ . 06

## Section-C

Q.9(i) Find the work done in moving a particle round the circle  $x^2 + y^2 = 9$ , in the xy-plane if the field of force is  $\vec{F} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$ . 02

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(ii) Define Solenoidal vector and Irrotational vector. 02

(iii) Define the centre of gravity of a solid occupying volume  $V$  having density  $\rho = f(x, y, z)$  02

(iv) Evaluate

$$\int_0^{\pi/2} \int_0^{\cos\theta} r\sqrt{a^2 - r^2} dr d\theta$$
 02

(v) Evaluate the double integral

$$\int_0^1 \int_0^1 (x+2) dy dx$$
 02

(vi) Find the area bounded by the curves  $x = y^2, x + y - 2 = 0$ . 02

(vii) Find the gradient of the function  $\varphi = y^2 - 4xy$  at  $(1,2)$  02

(viii) Find curl  $\vec{F}$  where  $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$  02

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