PC-4302/MH

A/2051

PARTIAL DIFFERENTIAL EQUATIONS–5 (ii)

(Semester–II)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all selecting *two* questions each from Section A and Section B and compulsory question of Section C.

SECTION–A

I. (a) Form partial differential equation by eliminating the arbitrary function from the equation :

$$f(x^2 + y^2 + z^2, x + y + z) = 0.$$ 3

(b) Solve : $x(y - z)p + y(z - x)q = z(x - y)$. 3

II. (a) Find the general solution of the following partial differential equation :

$$(x^2 - y^2 - z^2)\ p + 2xyq = 2xz.$$ 3

(b) Find the complete integral of $z^2 (p^2 + q^2) = x^2 + y^2$. 3

III. Classify and reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. 6

IV. Find the complete solution of the partial differential equation $2xz - px^2 - 2qxy + pq = 0$ by using Charpit's method. 6

SECTION–B

V. (a) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = e^y$. 3

(b) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$. 3

VI. A string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x, t)$. 6

VII. A rod of length l with insulated sides is initially at a uniform temperature. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$. 6

VIII. A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by

$u(x, 0) = 100 \sin \frac{\pi x}{8}, 0 < x < 8,$ while the two long edges

$x = 0$ and $x = 8$ as well as the other short edge are kept at 0°C, show that the steady-state temperature at any point of

the plane is given by $u(x, y) = 100e^{-\pi y/8} \sin \frac{\pi x}{8}$. 6

SECTION–C

(Compulsory Question)

IX. (a) Find the partial differential equation of all spheres of radius 5 and having their centres in the *xy*-plane.

(b) Find the general solution of $yzp + zxq = xy$.

(c) Show that the equation $z_{xx} + 2xz_{xy} + (1 - y^2) z_{yy} = 0$ is elliptic for all values of x, y in the region $x^2 + y^2 < 1$, parabolic on the boundary and hyperbolic outside the region.

(d) Find the equation of the surface which cuts orthogonally the family of spheres $x^2 + y^2 + z^2 = cy$, $c \neq 0$ arbitrary constant, and passes through the circle $z = 1, x^2 + y^2 = 4$.

(e) Find the general solution of the partial differential equation :

$$\frac{\partial^3 z}{\partial x^3} - 3\,\frac{\partial^3 z}{\partial^2 x \partial y} + 4\,\frac{\partial^2 z}{\partial y^3} - = e^{x+2y}.$$

(f) Solve : $\frac{\partial^2 z}{\partial x^2} + (a+b)\,\frac{\partial^2 z}{\partial x \partial y} + ab\,\frac{\partial^2 z}{\partial y^2} = xy.$

(g) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y).$

(h) Solve : $z = px + qy + p^2 + q^2$. (2×8=16)