G=10/2051

Sem-V

5928/MJ

BMH 501: Algebra II

Preponement (Dec-18)

Time Allowed: 3 Hours Maximum Marks: 75

Note: - Attempt any two questions each from section-A and section-B. Section-C is compulsory.

Section-A

$2\times 11.25=22.5$

Q.1. (a) If N is a normal subgroup of a group G and $N \cap G' = \{e\}$ then show that $N \subseteq Z(G)$.

(b) Show that S_n is not solvable for $n > 4$.

Q.2. State and prove Schreier's refinement theorem.

Q.3. a) Prove or disprove that if ring with unity such that $(xy)^3 = x^3y^3$ then R is commutative.

b) Verify Jordan Holder theorem for Quaternion group.

Q.4. Prove that every maximal ideal in a commutative ring with unity is a prime ideal. Also show that converse of this statement is not true.

Section-B

$2\times 11.25 = 22.5$

Q.5. State and prove second theorem of Ring Isomorphism.

Q.6. Prove that every ring can be embedded in a ring of endomorphism of some additive abelian group.

Q.7. Prove that every Euclidean Domain is a PID. Also show that the converse of this is not true.

Q.8. (a) Show that every irreducible element of UFD is prime element.

(b) Show that $Z[\sqrt{-6}]$ is not UFD.

Section-C

$3\times 10 = 30$

Q.9.

a) Prove that a group G is commutative iff $G' = \{e\}$.

b) Write down all composition series of cyclic group of order 24.

c) Distinguish Normal and subnormal series for a group.

d) The union of two ideals may or may not be an ideal. Justify this statement.

e) Give an example of a no commutative ring R and an ideal I of R such that quotient R/I is a field.

f) Show that kernel of a ring homomorphism is an ideal of ring.

g) Explain Field of quotients and embedding of a ring.

h) Show that $Z[\sqrt{-2}]$ is an ED.

i) Show that the element $1+i$ is irreducible in $Z[i]$.

j) Find all associates of $\bar{2}$ in ring $Z/(8)$.