AS-2051
CALCULUS-II PAPER -IV
SEMESTER -II

TIME :3 HOURS **M:M: 40**

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

706/MH

SECTION-A

I(a) Evaluate $\iint xy\, dx\, dy$ over the region A,

where A is the region common to the circles $x^2 + y^2 = x, x^2 + y^2 = y$. (3)

I(b) Evaluate by changing the order of integration $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dy\, dx}{\sqrt{y^4 - a^2x^2}}$. (3)

II(a)By changing into polar co-ordinates, evaluate $\iint \sqrt{\frac{36-4x^2-9y^2}{36+4x^2+9y^2}}\, dx\, dy$

over the region bounded by ellipse $4x^2 + 9y^2 = 36$ and the coordinate

axes lying in first quadrant. (3)

II(b) Find the area enclosed by the parabolas $x = 2y - y^2$ and $x = y^2$ (3)

III(a) Evaluate $\iiint \frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}\, dx\, dy\, dz$, over V,

where $V = \{(x, y, z) | x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1\}$. (3)

III(b) Show that volume bounded by cylinder $x^2 + y^2 = 4$ and

planes $y + z = 4, z = 0\ is\ 16\pi$. (3)

IV Find the centre of gravity of the uniform hemi-spherical region

with mass density $\mu(x, y, z) = x^2$. (6)

SECTION-B

V(a) Show that dot product of two non-zero vectors vanishes if and only

if they are perpendicular. (3)

V(b) Find unit vector perpendicular to both $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (3)

VI(a) Find equation of plane passing through line of intersection of the planes

$\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$ and $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + 4 = 0$ and parallel to x-axis. (3)

VI(b) Find the equation of the line(vector and cartesian both) which is parallel

To the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ and which passes through the point $(2, 3, 1)$ (3)

VII State and prove Stoke's Theorem. (6)

VIII(a) Verify divergence theorem for $\vec{A} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the

region bounded by $x^2 + y^2 = 4, z = 0\ and\ z = 3$. (3)

VIII(b) Evaluate by Green's Theorem in plane for

$\oint[(\cos x \sin y - xy)dx + \sin x \cos y \, dy]$ over curve C,

Where C is the circle $x^2 + y^2 = 1$. (3)

SECTION-C

IX(a) If a region A is defined as A = {(x, y): $0 \leq x \leq 3, 2 \leq y \leq 5$},

then show that $108 \leq \iint(2x^2 + 3y^2)dx\,dy \leq 837$.

IX(b) Find moment of inertia of the circular region A = {(x, y): $x^2 + y^2 \leq a^2$}

with unit density about x-axis.

IX(c) Evaluate $\iint r^3 dr d\theta$ *over the area included between the*

circles $r = 2\cos\theta$ *and* $r = \cos\theta$.

IX(d) Evaluate $\iiint(z^5 + z)\,dxdydz$ *over region*

$V = \{(x, y, z): x^2 + y^2 + z^2 \leq 1\}$

IX(e) Find the total work done in moving a particle in a force field given by

$\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ *to* $t = 2$.

IX(f) Define the flux of a vector point function across a surface.

IX(g) Find vector equation of plane which is at distance of 5 units from the origin

and which is normal to the vector $2\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$.

IX(h) Find the sine of angle between the vectors $\vec{a} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

706/MH (2× 8 = 16)