AS-2051
PARTIAL DIFFERENTIAL EQUATION -V
SEMESTER -II

TIME :3 HOURS **M:M: 40**

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

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SECTION-A

I (a). Find the partial differential equation of all spheres of radius 10 and centres lying on

Plane $x + y = 0$ (3)

I(b) Form a partial differential equation by elimination of arbitrary function from following relation: $xy - z^2 = f\left(\frac{x}{y}\right)$ (3)

II (a). Find the equation of the integral surface of the partial differential equation

$xp + yq = z$, which passes through the curve $x + y = 1, yz = 1$ (3)

II(b) Find the equation of the family of surfaces which cut orthogonally the cones of family

$x^2 + y^2 + z^2 = \lambda xy, \quad where\ \lambda\ is\ parameter.$ (3)

III(a) Solve $xr + p = 27x^2y^3, x > 0.$ (3)

III(b)Find singular solution of the following partial differential equation

$p^2 - q^2 = x - y$ (3)

IV Using Charpit's method to find complete solution of the following partial differential equation: $z^2 = pqxy.$ (6)

SECTION-B

V (a) Find the general solution of the partial differential equation

$2r - s - 3t = 10\,\frac{e^x}{e^y}$ (3)

V (b) Find the general solution of the partial differential equation

$r - 4s + 3t = \sqrt{(x + y)}$ (3)

VI (a) Find the general solution of the partial differential equation

$r + s - 6t = y\cos x$ (3)

VI (b) Find the general solution of the partial differential equation

$(D_x + D_y - 1)(D_x + D_y - 3)(D_x + D_y)z = e^{x+y}\sin(x + 2y)$ (3)

VII. Obtain the general solution of one dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

and also find the particular solution for which $u = f(x), \frac{\partial u}{\partial t} = g(x)\ at\ t = 0$ (6)

VIII Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ subject\ to$

$u(x,0) = u(x,m) = o\ where\ 0 \le x \le l\ \ and\ \ u(0,y) = 0,\ u(l,y) = F(y)$

$where\ 0 \le y \le m.$ (6)

SECTION-C

IX(a)Define the Lagranges's Linear Equation.

IX(b) State the conditions under which the partial differential equation

$$rR + sS + tT + f(x,y,z,p,q) = 0$$

To be classified as Parabolic, Elliptic or Hyperbolic. where R, S and T are continuous functions of x and y

IX(c) Find the general solution of the partial differential equation

$$ar = xy, \quad \text{where } r = \frac{\partial^2 z}{\partial x^2}$$

IX(d) Find the complete solution of the partial differential equation

$$(p-q)(z-px-qy) = 1$$

IX(e). Solve the partial differential equation

$$r - s - 2t + 2p + 2q = 0$$

IX(f) Find Particular Integral(PI) of the partial differential equation

$$r - t - 3p + 3q = 54xy$$

IX(g) Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection

$$f(x) = x - x^2$$

IX(h) Write a short note on Method of separation of variables for solving second order partial differential equation of the form

$pP + qQ + rR + sS + tT = W$, where P,Q,R,S,T,W are functions of x, y only

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$2\times 8 = 16$